

Year 10 Curriculum Overview Maths 2023-24

Year 10	Term 1			
Unit Title	Congruence, Similarity and Enlargement	Trigonometry	Represent Solutions of Equations and Inequalities	Simultaneous Equations
Approximate Number of Lessons	10.5	10.5	10.5	10.5
Curriculum Content	<ul style="list-style-type: none"> Enlarge a shape by a positive integer scale factor. Enlarge a shape by a fractional scale factor. Enlarge a shape by a negative scale factor. Identify similar shapes. Works out missing sides and angles in a pair of given similar shapes. Use parallel line rules to work out missing angles. Establish that a pair of triangles are similar. Explore areas of similar shapes. Explore volumes of similar shapes. Solve mixed problems involving similar shapes. Understand the difference between congruence and similarity. Understand and use conditions for congruent triangles Prove a pair of congruent triangles are equal. 	<ul style="list-style-type: none"> Explore ratio in similar right-angled triangles. Work fluently with the hypotenuse, opposite and adjacent sides. Use the Sine, Cosine and Tangent ratio to find missing side lengths. Use the Sine, Cosine and Tangent ratio to find missing angles. Calculate sides in right angle triangles using Pythagoras Theorem. Select an appropriate method to solve right angle triangle problems. Use Trigonometry in 3D shapes Use the formula $\frac{1}{2}ab\sin C$ to find the area of a triangle. Understand and use the Sine Rule to find missing lengths and angles. Understand and use the Cosine Rule to find missing lengths and angles. Choose and use the Sine and Cosine Rules. 	<ul style="list-style-type: none"> Understand the meaning of a solution. Form and solve 1 and 2 step equations Form and solve 1 and 2 step inequalities. Show solutions to inequalities on a number line. Interpret representations on number lines as inequalities. Represent solutions to inequalities using set notation. Draw straight line graphs Find solutions to equations using straight line graphs. Represent solutions to single and multiple inequalities on a graph. Form and solve equations with unknown on both sides Form and solve inequalities with unknown on both sides. Form and solve more complex equations and inequalities. Solve quadratic equations by factorisation. Solve quadratic inequalities in one variable. 	<ul style="list-style-type: none"> Understand that equations can have more than one solution Determine whether a given (x, y) is a solution to a pair of simultaneous equations. Solve a pair of linear simultaneous equations by substituting a known variable, substituting an expression, using graphs or adding/subtracting equations Solve a pair of linear simultaneous equations by adjusting one and / or both equations. Solve a pair of linear simultaneous equations from given information. Form and solve a pair of linear simultaneous equations from given information. Determine whether a given (x, y) is a solution to both a linear and quadratic equation. Solve a pair of simultaneous equations (one linear, one quadratic) using graphs. Solve a pair of simultaneous equations (one linear, one quadratic) algebraically. Solve a pair of simultaneous equations involving a third unknown.
Links to prior learning	<ul style="list-style-type: none"> Enlargement of 2D shapes. Similarity of 2D shapes. Scale factors. Rules for angles in parallel lines. 	<ul style="list-style-type: none"> Enlargement of 2D shapes. Similarity of 2D shapes. Scale factors Rules for angles in parallel lines. 	<ul style="list-style-type: none"> Solve linear equations Algebraic substitution Plot (x, y) coordinates Factorise quadratic expressions 	<ul style="list-style-type: none"> Solve linear equations Algebraic substitution Draw linear and quadratic graphs
Cultural Capital Opportunities	How do architects use scale?	How is trigonometry used? Is it really Pythagoras' Theorem?	Babylonian Equations	Euclid's first axiom
Assessment Focus	Edexcel past paper non-calculator, lasting 1 hour 30 minutes		Edexcel past paper calculator, lasting 1 hour 30 minutes	
Knowledge Organiser	Unit 01 - Congruence, similarity & enlargement	Unit 02 - Trigonometry	Unit 03 - Representing solutions of equations and inequalities	Unit 04 - Simultaneous Equations

Year 10 Curriculum Overview Maths 2023-24

Year 10	Term 2					
Unit Title	Angles and bearings	Working with circles	Vectors	Ratios and fractions	Percentages and interest	Probability
Approximate Number of Lessons	7	7	7	7	7	7
Curriculum Content	<ul style="list-style-type: none"> Use cardinal directions and related angles Draw and interpret scale diagrams Understand and represent bearings Measure and read bearings Make scale drawings using bearings Calculate bearings using angle rules Solve bearings problems using Pythagoras and trigonometry Solve bearing problems using the Sine and Cosine rules 	<ul style="list-style-type: none"> Recognise and label parts of a circle Calculate fractional parts of a circle Calculate the length of an arc Calculate the area of a sector Investigate circle theorems Understand and use the volume of a cylinder, cone and sphere Understand and use the surface area of a cylinder, cone and sphere Solve area and volume problems involving similar shapes 	<ul style="list-style-type: none"> Understand and represent vectors Use and read vector notation Draw and understand vectors multiplied by a scalar Draw and understand addition and subtraction of vectors Explore vector journeys Explore quadrilaterals using vectors Understand parallel vectors Explore collinear points using vectors Use vectors to construct geometric arguments and proofs 	<ul style="list-style-type: none"> Compare quantities using a ratio Link ratios and fractions Share in a ratio Use ratios and fractions to make comparisons Link ratios and graphs Solve problems with currency conversion Link ratio and scales Use and interpret ratios of the form $1:n$ and $n:1$ Solve 'best buy' problems Combine a set of ratios Link ratio and algebra Ratio in area and volume problems 	<ul style="list-style-type: none"> Convert and compare fractions, decimals and percentages Work out percentages of amounts with and without a calculator Increase and decrease by a given percentage Express one number as a percentage of another Calculate simple and compound interest Repeated percentage change Find the original amount after a percentage change Solve problems involving growth and decay Understand iterative processes Solve problems with percentages, ratios and fractions 	<ul style="list-style-type: none"> Know how to add, subtract and multiply fractions Find probabilities using equally likely outcomes Use the property that probabilities sum to 1 Using experimental data to estimate probabilities Find probabilities from tables, Venn diagrams and frequency trees Construct and interpret sample spaces for more than one event Calculate probability with independent events Use tree diagrams for independent and dependent events Construct and interpret conditional probabilities in tree diagrams, Venn diagrams and two-way tables
Links to prior learning	<ul style="list-style-type: none"> Use cardinal directions and related angles Scale diagrams 	<ul style="list-style-type: none"> Parts of a circle Substitution 	<ul style="list-style-type: none"> Changing from ratio to fractions Collecting like terms 	<ul style="list-style-type: none"> Link ratios, fractions and graphs Share in a given ratio 	<ul style="list-style-type: none"> Percentages of amounts Increasing and decreasing by a percentage 	<ul style="list-style-type: none"> Operating with fractions Equally likely outcomes
Cultural Capital Opportunities	Navigation technology	Circle secrets	Physics for Kids: Scalars and Vectors Play a game of chess using vectors	Activity: Currency Conversion Find the cost of an i-Phone in 6 different currencies and convert the price to £	The Mandelbrot set	Probability in Real Life The Risk of Everyday Activities
Assessment Focus	Edexcel past paper calculator, lasting 1 hour 30 minutes			Edexcel past paper non-calculator, lasting 1 hour 30 minutes		
Knowledge Organiser	Unit 05 - Angles and bearings	Unit 06 - Working with circles	Unit 07 - Vectors	Unit 08 - Ratios and fractions	Unit 09 - Percentages and Interest	Unit 10 - Probability

Year 10 Curriculum Overview Maths 2023-24

Year 10	Term 3			
Unit Title	Collecting, representing and interpreting data	Non-calculator methods	Types of numbers and sequences	Indices and roots
Approximate Number of Lessons	21	7	7	7
Curriculum Content	<ul style="list-style-type: none"> Understand populations and samples Construct a stratified sample Primary and secondary data Construct and interpret frequency tress and frequency polygons Construct and interpret two-way tables Construct and interpret line and bar charts, including composite bar charts Construct and interpret pie charts Criticise charts and graphs Construct and interpret histograms Find and interpret averages from a list and table Construct and interpret stem-and-leaf diagrams Construct and interpret cumulative frequency diagrams Use cumulative frequency diagrams to find measures Construct and interpret box plots Compare distributions using charts and measures Compare distributions using complex charts and measures Construct and interpret scatter graphs Draw and use a line of best fit Understand extrapolation 	<ul style="list-style-type: none"> Mental/written methods of integer/decimal addition, subtraction multiplication and division The four rules of fraction arithmetic Exact answers Rational and irrational numbers, and converting recurring decimals Understand and use surds Calculate with surds Rounding to decimals and significant figures Estimating answers to calculations Understand and use limits of accuracy Upper and lower bounds Solve financial maths problems 	<ul style="list-style-type: none"> Understand the difference between factors and multiples Understand primes and express a number as a product of its prime factors Find the HCF and LCM of a set of numbers Describe and continue arithmetic and geometric sequences Explore other sequences Describe and continue sequences involving surds Find the rule for the n^{th} term of a linear sequence Find the rule for the n^{th} term of a quadratic sequence 	<ul style="list-style-type: none"> Square and cube numbers Calculate higher powers and roots Powers of ten and standard form The addition and subtraction rules for indices Understand and use the power zero and negative indices Work with powers of powers Understand and use fractional indices Calculate with numbers in standard form
Links to prior learning	<ul style="list-style-type: none"> Two-way tables Pie charts Averages Time series graphs Scatter graphs Lines of best fit 	<ul style="list-style-type: none"> Written methods for four operations Rounding and estimating Operating with fractions 	<ul style="list-style-type: none"> Describe and continuing sequences Factors and multiples Linear sequences 	<ul style="list-style-type: none"> Squaring and square rooting Write numbers in standard form
Cultural Capital Opportunities	Compare the data from the 2021 census to the data of the 1971/1921 census Office for National Statistics	Financial Mathematics	The Fibonacci Sequence & The Golden Ratio	Real Life Applications of Indices & Powers
Assessment Focus	None	Two Edexcel past papers calculator, lasting 1 hour 30 minutes each		
Knowledge Organiser	Unit 11 - Collecting, representing and interpreting	Unit 12 - Non-calculator methods	Unit 13 - Types of number & sequences	Unit 14 - Indices & Roots

YEAR 10 — SIMILARITY...

Congruence, similarity & enlargement

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Centre of enlargement: the point the shape is enlarged from

Similar: when one shape can become another with a reflection, rotation, enlargement or translation

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

Parallel: straight lines that never meet (equal gradients)

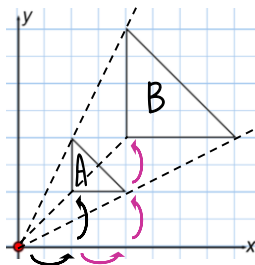
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

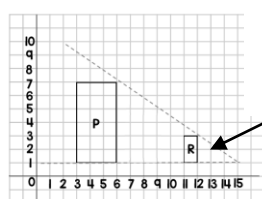
The distance from the point enlarges by 2



Fractional scale factors R

Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



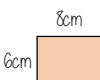
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180°

Similar shapes

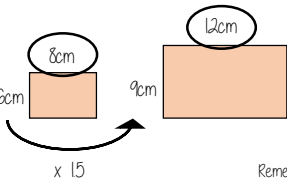


Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: $6:9$ and $8:12$
 $2:3$ and $2:3$

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

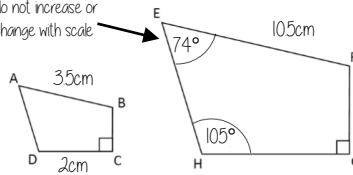
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

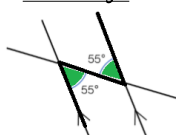
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines R

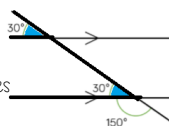
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

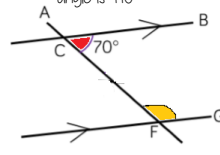
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

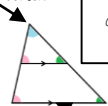


As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

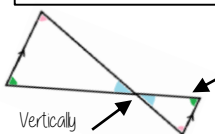
Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size



Parallel lines — all angles will be the same in both triangle

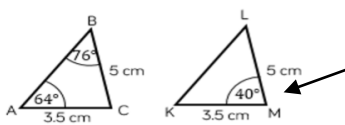
As all angles are the same this is similar — it only one pair of sides are needed to show equality



All the angles in both triangles are the same and so similar

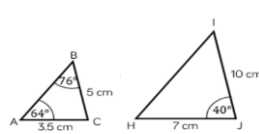
Congruence and Similarity

Congruent shapes are identical — all corresponding sides and angles are the same size



$\angle C \hat{= } \angle M$

Because all the angles are the same and $AC=KM$ $BC=LM$ triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 10 — SIMILARITY...

Trigonometry

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement

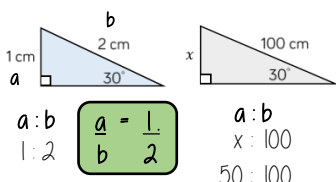
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

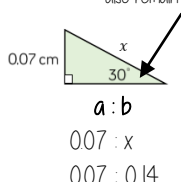
Inverse: function that has the opposite effect.

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

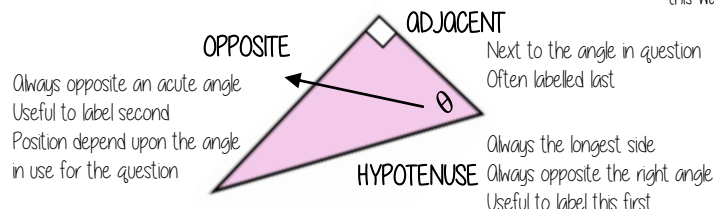


When the angle is the same the ratio of sides a and b will also remain the same



Hypotenuse, adjacent and opposite

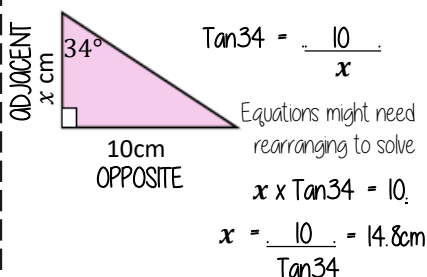
ONLY right-angled triangles are labelled in this way



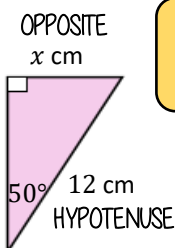
Tangent ratio: side lengths

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula



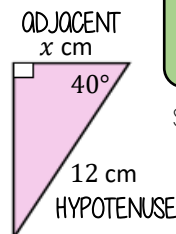
Sin and Cos ratio: side lengths



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

NOTE

The $\sin(x)$ ratio is the same as the $\cos(90-x)$ ratio



$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

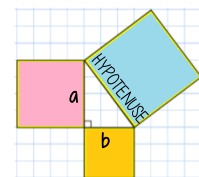
Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem

R

$$\text{Hypotenuse}^2 = a^2 + b^2$$



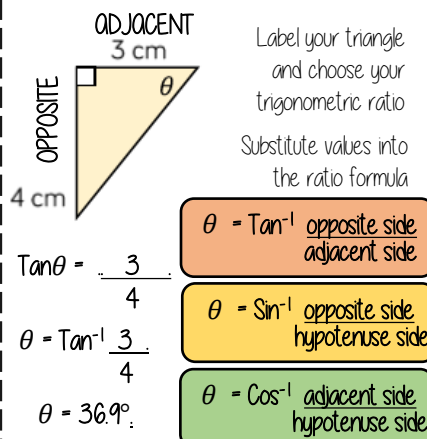
This is commutative — the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras

- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

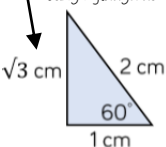
Sin, Cos, Tan: Angles

Inverse trigonometric functions



Key angles

This side could be calculated using Pythagoras



$$\tan 30 = \frac{1}{\sqrt{3}}$$

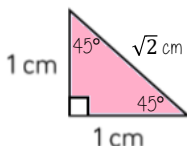
$$\tan 60 = \sqrt{3}$$

$$\cos 30 = \frac{\sqrt{3}}{2}$$

$$\cos 60 = \frac{1}{2}$$

$$\sin 30 = \frac{1}{2}$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$



$$\tan 45 = 1$$

$$\cos 45 = \frac{1}{\sqrt{2}}$$

$$\sin 45 = \frac{1}{\sqrt{2}}$$

Key angles 0° and 90°

$$\tan 0 = 0$$

$$\tan 90$$

This value cannot be defined — it is impossible as you cannot have two 90° angles in a triangle



$$\sin 0 = 0$$

$$\sin 90 = 1$$

$$\cos 0 = 1$$

$$\cos 90 = 0$$

YEAR 10 — DEVELOPING ALGEBRA...

Representing solutions of equations and inequalities

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Form and solve equations and inequalities
- Represent and interpret solutions on a number line as inequalities
- Draw straight line graphs and find solutions to equations
- Form and solve equations and inequalities with unknowns on both sides

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal — it will have an equals sign =

Expression: numbers, symbols and operators grouped together to show the value of something

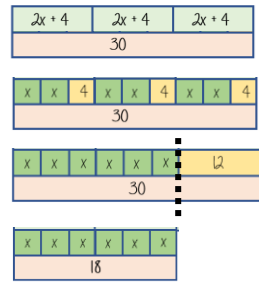
Identity: An equation where both sides have variables that cause the same answer includes \equiv

Linear: an equation or function that is the equation of a straight line

Intersection: the point that two lines meet

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations R



$$3(2x + 4) = 30$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

-12

-12

Substitute to check your answer.
This could be negative or a fraction or decimal

$$6x = 18$$

-6

-6

$$x = 3$$

Form and solve inequalities R



Two more than treble my number is greater than 11

Form

$$x \rightarrow x3 \rightarrow +2 \rightarrow 11$$

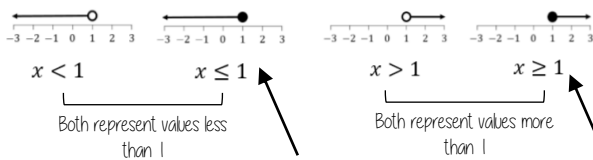
$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

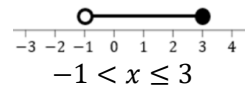
$$x > 3$$

Solutions on a number line



- Includes the value it sits above
- Does NOT include the value it sits above

Values less than or equal to 3 but also more than -1



This includes the integer values 0, 1, 2, 3

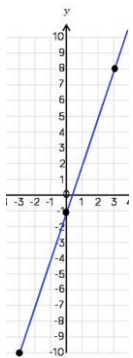
Plotting straight line graphs R

$$y = 3x - 1$$

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

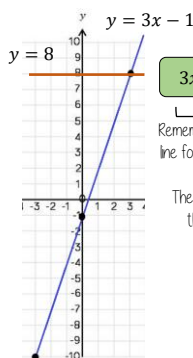
Find solutions graphically

For linear equations there is only one point the graph meets the x value

$$x = 2$$

$$y = 4$$

These two lines will cross at (2,4) because they are just $x=$ and $y=$ they are parallel to axes and meet in one place

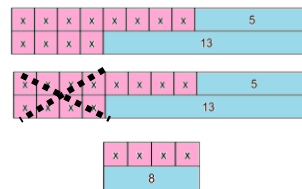


Remember equation of a line format is $y = mx + c$

The solution is the point the two lines meet **(3,8)**

Equations: unknown on both sides R

$$8x + 5 = 4x + 13$$



$$8x + 5 = 4x + 13$$

$$-4x \quad -4x$$

$$4x + 5 = 13$$

$$-5 \quad -5$$

$$4x = 8$$

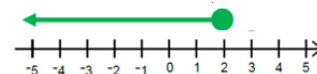
$$\div 4 \quad \div 4$$

$$x = 2$$

Inequalities: unknown on both sides

$$8x + 5 \leq 4x + 13$$

$$x \leq 2$$



Only value 2 or less will satisfy this inequality

YEAR 10 — DEVELOPING ALGEBRA...

Simultaneous Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Determine whether (x,y) is a solution
- Solve by substituting a known variable
- Solve by substituting an expression
- Solve graphically
- Solve by subtracting/ adding equations
- Solve by adjusting equations
- Form and solve linear simultaneous equations

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet

Equation: an equation says that two things are equal — it will have an equals sign =

Substitute: replace a variable with a numerical value

LCM: lowest common multiple (the first time the times table of two or more numbers match)

Eliminate: to remove

Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)

Coordinate: a set of values that show an exact position

Intersection: the point two lines cross or meet

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line $y=3x+5$?

This coordinate represents $x=1$ and $y=8$

$$y = 3x + 5$$

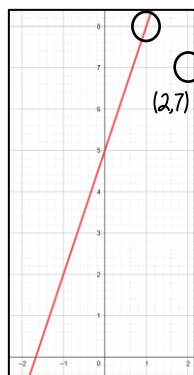
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate (1,8) IS on the line $y=3x+5$

Is (2,7) on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal $6+5$



Substituting known variables

A line has the equation $3x + y = 14$

Two different variables, two solutions

Stephanie knows the point $x = 4$ lies on that line. Find the value for y

$$x = 4$$

$$3x + y = 14$$

$$3(4) + y = 14$$

$$12 + y = 14$$

$$-12 \quad -12$$

$$y = 2$$

Substituting in an expression

$$x = 2y$$

$$x + y = 30$$

Pair of simultaneous equations (two representations)

Substitute $2y$ in place of the x variable as they represent the same value

$$x = 2y$$

$$x + y = 30$$

$$3y = 30$$

$$3y = 30$$

$$\div 3 \quad \div 3$$

$$y = 10$$

$$x = 2y$$

$$x = 20$$

Solve graphically

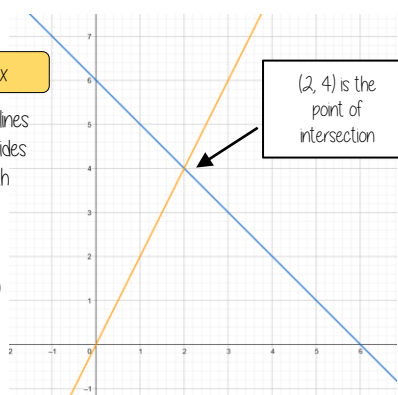
$$x + y = 6$$

$$y = 2x$$

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$$x = 2 \text{ and } y = 4$$



Solve by subtraction

$$18$$

$$10$$

$$8$$

$$x = 4$$

$$y = 3$$

$$3x + 2y = 18$$

$$x + 2y = 10$$

$$2x = 8$$

$$\div 2 \quad \div 2$$

$$x = 4$$

$$x + 2y = 10$$

$$(4) + 2y = 10$$

$$-4 \quad -4$$

$$2y = 6$$

$$\div 2 \quad \div 2$$

$$y = 3$$

$$x + x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x + x + y + y = 18$$

$$x + y + y = 10$$

$$x + x = 8$$

$$x = 4$$

$$y = 3$$

Solve by addition

Addition makes zero pairs

$$3x + 2y = 16$$

$$+ 6x - 2y = 2$$

$$9x = 18$$

$$\div 9 \quad \div 9$$

$$x = 2$$

$$3x + 2y = 16$$

$$3(2) + 2(y) = 16$$

$$6 + 2y = 16$$

$$-6 \quad -6$$

$$2y = 10$$

$$y = 5$$

$$x + x + x + y + y = 16$$

$$x + x + x + y + y = 2$$

$$x + x + x = 18$$

$$x = 2$$

$$y = 5$$

Solve by adjusting one

$$h + j = 12 \quad \text{No equivalent values}$$

$$2h + 2j = 29$$

$$2h + 2j = 24$$

$$2h + 2j = 29$$

By proportionally adjusting one of the equations — now solve the simultaneous equations choosing an addition or subtraction method

$$12$$

$$29$$

$$24$$

$$29$$

Solve by adjusting both

$$2x + 3y = 39$$

$$5x - 2y = -7$$

Use LCM to make equivalent x OR y values

Because of the negative values using zero pairs and y values is chosen choice

$$4x + 6y = 78$$

$$15x - 6y = -21$$

Now solve by addition

$$4x + 6y = 78$$

$$15x - 6y = -21$$

Addition makes zero pairs

YEAR 10 — GEOMETRY...

@whisto_maths

Angles and bearings

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent bearings
- Measure and read bearings
- Make scale drawings using bearings
- Calculate bearings using angle rules
- Solve bearings problems using Pythagoras and trigonometry

Keywords

Cardinal directions: the directions of North, South, East, West

Angle: the amount of turn between two lines around their common point

Bearing: the angle in degrees measured clockwise from North

Perpendicular: where two lines meet at 90°

Parallel: straight lines always the same distance apart and never touch. They have the same gradient

Clockwise: moving in the direction of the hands on a clock

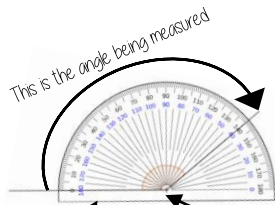
Construct: to draw accurately using a compass, protractor and or ruler or straight edge

Scale: the ratio of the length of a drawing to the length of the real thing

Protractor: an instrument used in measuring or drawing angles

Measure angles to 180°

R



The base line follows the line segment

Make sure the cross is at the point the two lines meet

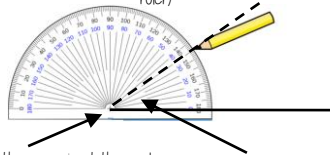
Read from 0° on the base line. Remember to use estimation. This is an obtuse angle so between 90° and 180°

Draw angles up to 180°

R

Draw a 35° angle

Make a mark at 35° with a pencil. And join to the angle point (use a ruler)

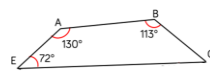


Make sure the cross is at the end of the line (where you want the angle)

The angle

Angle notation

The letter in the middle is the angle. The arc represents the part of the angle



Angle Notation: three letters $\angle ABC$. This is the angle at $B = 113^\circ$

$\angle ABC$ is also used to represent the angle at B

Scale drawings

R

1 : 20

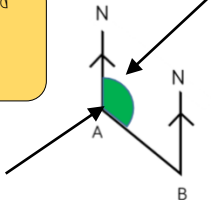
For every 1cm on the model there are 20cm in real life

Remember: Scale drawings **ONLY** change lengths and distances. Angles remain the same

Understand and represent bearings

- A bearing is always measured from **NORTH**
- It is always given as three figures

The bearing of B from A is calculated by measuring the highlighted angle



The angle indicated starts from the North line at A and joins the path connecting A to B

This angle shows the bearing of B from A

The sentence... "Bearing of ____ from ____" is really important in identifying the bearing being represented

Using **estimation** it is clear this angle is between 090° and 180°

Directions



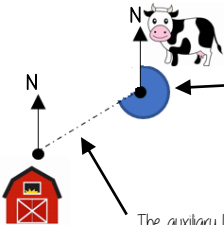
Clockwise



Anti-Clockwise



Measure and read bearings



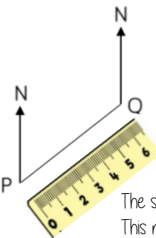
The bearing of the cow to the barn

This angle is measured from **NORTH**. It is measured in a clockwise direction. **Estimation** indicates this angle is between 180° and 270° . Use a protractor to measure accurately. Remember: bearings are written as three figures.

The auxiliary line is drawn to help you measure and draw the angle that is measured to represent the bearing

Scale drawings using bearings

Remember — angles **DO NOT** change size in scaled drawings



The bearing measurements do not change from "real life" to images

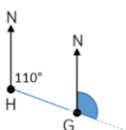
The units in the ratio scale are the same

The scale may need to be calculated from the image. This represents 30km from P to Q

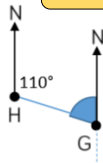
6cm = 30km
6:30,000,000

Bearings with angle rules

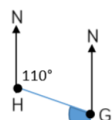
Because two North lines are **PARALLEL**....



They form **corresponding angles** and therefore are the same size



They form **co-interior angles** and add up to 180°



They form **alternate angles** and therefore are the same size

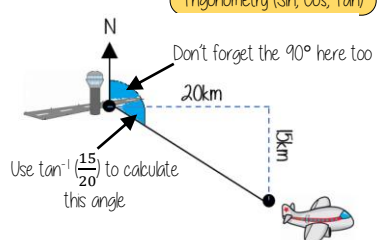
Bearings with right-angled geometry

Look for Right-angles. Pythagoras. Trigonometry (Sin, Cos, Tan)

"Due West" bearing of 270° makes a 90° angle

"Due East" bearing of 090° makes a 90° angle

A plane flies East for 20km then turns South for 15km. Find the bearing of the plane from where it took off



YEAR 10 — GEOMETRY...

@whisto_maths

Working with circles

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise and label parts of a circle
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- Understand and use volume of a cone, cylinder and sphere.
- Understand and use surface area of a cone, cylinder and sphere.

Keywords

Circumference: the length around the outside of the circle — the perimeter

Area: the size of the 2D surface

Diameter: the distance from one side of a circle to another through the centre

Radius: the distance from the centre to the circumference of the circle

Tangent: a straight line that touches the circumference of a circle

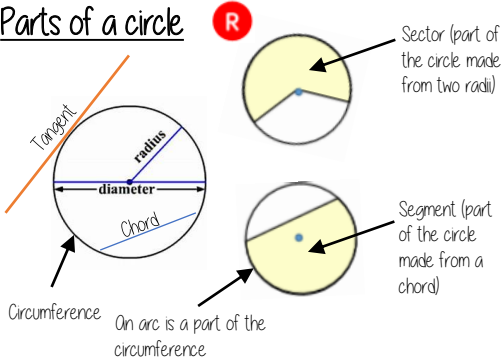
Chord: a line segment connecting two points on the curve

Frustum: a pyramid or cone with the top cut off

Hemisphere: half a sphere

Surface area: the total area of the surface of a 3D shape.

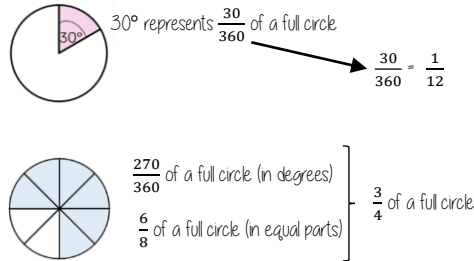
Parts of a circle



Fractional parts of a circle

A circle is made up of 360°

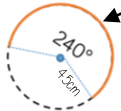
Formula to remember:
Area of a circle = πr^2
Circumference of a circle = πd or $2\pi r$



The fraction of the circle is as $\frac{\theta}{360}$
 θ represents the degrees in the sector

Arc length

Remember an arc is part of the circumference
Circumference of the whole circle = $\pi d = \pi \times 9 = 9\pi$



$$\text{Arc length} = \frac{\theta}{360} \times \text{circumference}$$

$$= \frac{240}{360} \times 9\pi = \frac{2}{3} \times 9\pi = 6\pi$$

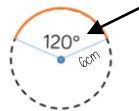
Perimeter

Perimeter is the length around the outside of the shape
This includes the arc length and the radii that enclose the shape

$$\text{Perimeter} = \frac{\theta}{360} \times \text{circumference} + 2r = 6\pi + 9$$

Sector area

Remember a sector is part of a circle
Area of the whole circle = $\pi r^2 = \pi \times 6^2 = 36\pi$



$$\text{Sector area} = \frac{\theta}{360} \times \text{area of circle}$$

$$= \frac{120}{360} \times 36\pi = \frac{1}{3} \times 36\pi = 12\pi$$

Volume of a cone and a cylinder

$$\text{Volume Cylinder} = \pi r^2 h$$

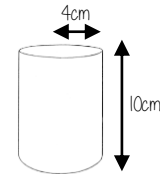


A cylinder is a prism — cross section is a circle



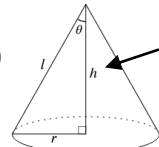
$$\text{Volume Cone} = \frac{1}{3} \pi r^2 h$$

A cone is a pyramid with a circular base



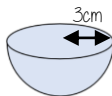
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 4^2 \times 10 \\ &= \pi \times 160 \\ &= 160\pi \text{ cm}^2 \end{aligned}$$

Give your answer in terms of π'
means NOT in terms of pi $\approx 502.7 \text{ cm}^2$



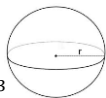
The height of a cone is the perpendicular height from the vertex to the base
Look out for trigonometry or Pythagoras theorem — the radius forms the base of a right-angled triangle

Volume of a sphere



$$\begin{aligned} \text{Volume Sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \times \pi \times 3^3 \\ &= \frac{4}{3} \times \pi \times 27 = 36\pi \end{aligned}$$

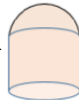
A hemisphere is half the volume of the overall sphere
 $= 36\pi \div 2 = 18\pi$



$$\text{Volume Sphere} = \frac{4}{3} \pi r^3$$

NOTE: This is now a cubed value

Look out for hemispheres being placed on other 3D shapes, e.g. cones and cylinders



Surface area of a sphere



Radius = 5cm

$$\begin{aligned} \text{Surface area} &= 4\pi r^2 \\ &= 4 \times \pi \times 5^2 \\ &= 4 \times \pi \times 25 \\ &= 100\pi \end{aligned}$$

The curved surface area of a sphere

$$\text{Surface area} = 4\pi r^2$$

A hemisphere has the curved surface AND a flat circular face



$$= 100\pi \div 2 = 50\pi$$

$$= 50\pi + \pi \times 5^2$$

$$\text{Hemisphere} = 75\pi$$

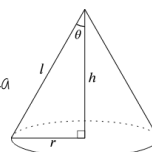
Surface area of cones and cylinders

$$\text{Surface area cylinder} = 2\pi r^2 + \pi d h$$



The area of two circles (top and bottom face) + the area of the curved face

The length of shape B is the circumference of the circles



$$\text{Curved surface area Cone} = \pi r l$$

Look out for the use of Pythagoras to calculate the length l

Total surface area = curved face + circle face (area of base)

YEAR 10 — GEOMETRY...

@whisto_maths

Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going

Magnitude: the magnitude of a vector is its length

Scalar: a single number used to represent the multiplier when working with vectors

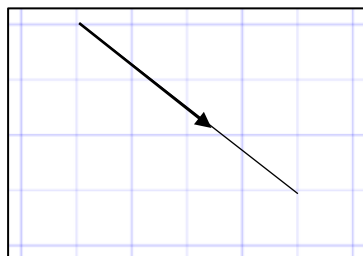
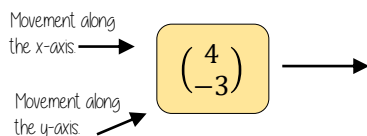
Column vector: a matrix of one column describing the movement from a point

Resultant: the vector that is the sum of two or more other vectors

Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another



Vectors show both direction and magnitude

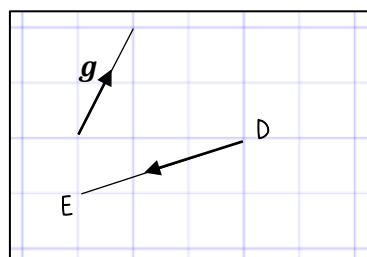
The arrow is pointing in the direction from starting point to end point of the vector.

The direction is important to correctly write the vector

The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)

The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation \overrightarrow{DE} is another way to represent the vector joining the point D to the point E

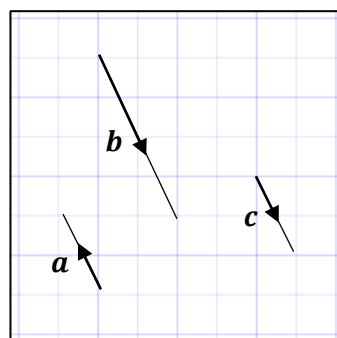
$$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so \mathbf{g} represents the vector $\mathbf{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$$\mathbf{b} = 2 \times \mathbf{c} = 2\mathbf{c}$$

Multiply \mathbf{c} by 2 this becomes \mathbf{b} .
The two lines are parallel

$$\mathbf{a} = -1 \times \mathbf{c} = -\mathbf{c}$$

The vectors \mathbf{a} and \mathbf{c} are also parallel. A negative scalar causes the vector to reverse direction

$$\mathbf{b} = -2 \times \mathbf{a} = -2\mathbf{a}$$

$$\mathbf{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

Addition of vectors

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

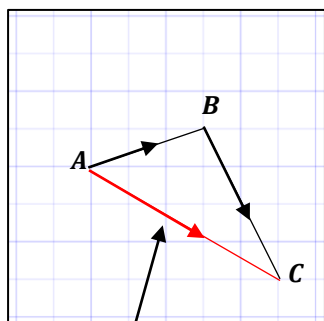
$$\overrightarrow{AB} + \overrightarrow{BC}$$

$$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

$$= \begin{pmatrix} 3+2 \\ 1+(-4) \end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Look how this addition compares to the vector \overrightarrow{AC}



The resultant

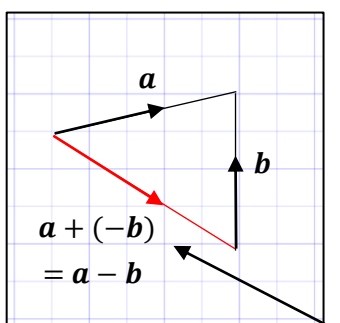
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

Addition and subtraction of vectors

$$\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 5 & -0 \\ 1 & -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$$



$$\mathbf{a} + (-\mathbf{b}) = \mathbf{a} - \mathbf{b}$$

The resultant is $\mathbf{a} - \mathbf{b}$ because the vector is in the opposite direction to \mathbf{b} which needs a scalar of -1

YEAR 10 — PROPORTION...

@whisto_maths

Ratios and fractions

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare quantities using ratio
- Link ratios and fractions and make comparisons
- Share in a given ratio
- Link Ratio and scales and graphs
- Solve problems with currency conversions
- Solve 'best buy' problems
- Combine ratios

Keywords

Ratio: a statement of how two numbers compare

Equivalent: of equal value

Proportion: a statement that links two ratios

Integer: whole number, can be positive, negative or zero

Fraction: represents how many parts of a whole

Denominator: the number below the line on a fraction. The number represent the total number of parts.

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken

Origin: (0,0) on a graph. The point the two axes cross

Gradient: The steepness of a line

Compare with ratio

'For every dog there are 2 cats'

Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given.

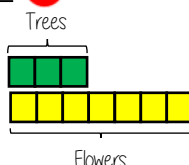
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have to be of the same value to compare ratios

Ratios and fraction

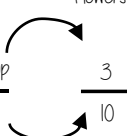
Trees: Flowers

3:7



Fraction of trees

Number of parts of in group
Total number of parts



Ratio

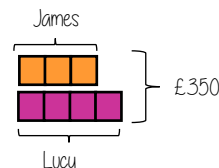
Fraction

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4
Work out how much each person earns

Model the Question

James: Lucy
3:4



Find the value of one part

Whole: £350

7 parts to share between (3 James, 4 Lucy)

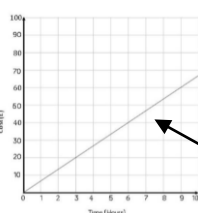
£350 ÷ 7 = £50
□ = one part = £50

Put back into the question

James = 3 × £50 = £150

Lucy = 4 × £50 = £200

Ratio and graphs



Graphs with a constant ratio are directly proportional

- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image: Real life
1cm: 30cm
10cm: 300cm



Conversion between currencies

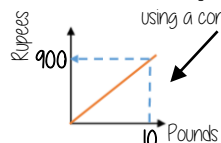
£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

£1 = 90 Rupees
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4:20
1:5

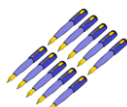
This side has to be divided by 4 too — to keep in proportion

the n part does not have to be an integer for this type of question

Best buys



4 pens costs £2.60



10 pens costs £6.00

*1 pen costs... £2.60 ÷ 4 = £0.65

£6.00 ÷ 10 = £0.60

*1-pound buys... 4 ÷ 2.60 = 1.54 pens

10 ÷ 6 = 1.67 pens

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost 'per pen'

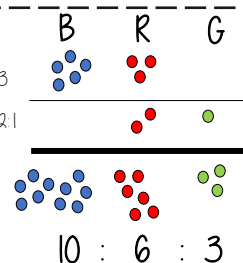
The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



Lowest common multiple of the ratio both statements share

Use equivalent ratios to allow comparison of the group that is common to both statements

YEAR 10 — PROPORTION...

Percentages and Interest

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Convert and compare FDP
- Work out percentages of amounts
- Increase/ decrease by a given percentage
- Express one number as a percentage
- Calculate simple and compound interest
- Calculate repeated percentage change
- Find the original value
- Solve problems with growth and decay

Keywords

Exponent: how many times we use a number in multiplication. It is written as a power.

Compound interest: calculating interest on both the amount plus previous interest.

Depreciation: a decrease in the value of something over time.

Growth: where a value increases in proportion to its current value such as doubling.

Decay: the process of reducing an amount by a consistent percentage rate over time.

Multiplier: the number you are multiplying by.

Equivalent: of equal value.

Compare FDP

Comparisons are easier in the same format.

70/100 → This also means 70 ÷ 100 → 70 out of 100 squares → 70 'hundredths' = 7 'tenths' 0.7 → 70 hundredths = 70%

Using a calculator → $\frac{70}{100} = 0.7$ → Convert to a decimal → × 100 converts to a percentage

Be careful of recurring decimals
e.g. $\frac{1}{3} = 0.333333$
 $\frac{1}{3} = 0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount

Find $\frac{3}{5}$ of £60 → £60 → £12 £12 £12 £12 £12 → £36

Remember $\frac{3}{5} = 60\% = 0.6$
10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5} = 60\% = 0.6$
60% of £60 = 0.6 × 60 = £36

Percentage increase/decrease

100% → 42% → Decrease by 58% → 100% - 58% = 42%
100 - 0.58 = 0.42 → Multiplier Less than 1

100% → 12% → Increase by 12% → 100% + 12% = 112%
100 + 0.12 = 112 → Multiplier More than 1

Express as a percentage

27 per every 50 shaded → $\frac{27}{50}$ → 54 per every 100 shaded → $\frac{54}{100}$ → 54%

$\frac{13}{30}$ → $\frac{13}{30} \times 100 = 43.333...%$ → 43%

Can't use equivalence easily to find 'per hundred' → Decimal percentages are still a percentage.

Simple and compound interest

Simple Interest
James invests £2000 at 5% simple interest → £2000 → £100 → The original value increases by this amount every year

Compound Interest
Tess invests £100 at 10% compound interest for 3 years → £100 → £110 → £121 → £132.10 → The multiplier 1.10 repeats each year

Repeated percentage change

Compound Interest
Tess invests £100 at 10% compound interest for 3 years → £100 → £110 → £121 → £132.10 → The multiplier 1.10 repeats each year

Depreciation
Depreciation calculations use multipliers less than 1
Multipliers are commutative — an overall multiplier effect can be calculated by combining the multipliers separately
e.g. Increase of 10% then a reduction of 10% → $1.10 \times 0.90 = 0.99$ → The multiplier

Growth and decay

Compound growth → Exponential growth graph

Compound decay → Exponential decay graph

Compound growth and compound decay are exponential graphs

Decay — the values get closer to 0
The constant multiplier is less than one

Growth — the values increase exponentially
The constant multiplier is more than one

Find the original value

Percentage calculations
Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test?
Original × 0.6 = 24 → 24 ÷ 0.6 = 40 marks
10% = 6
100% = 40
Total questions on test

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?
Original × 1.2 = 3000
120% = £3000
10% = £250
100% = £2500

YEAR 10 — PROPORTION...

@whisto_maths

Probability

What do I need to be able to do?

By the end of this unit you should be able to:

- Add, Subtract and multiply fractions
- Find probabilities using likely outcomes
- Use probability that sums to 1
- Estimate probabilities
- Use Venn diagrams and frequency trees
- Use sample space diagrams
- Calculate probability for independent events
- Use tree diagrams

Keywords

Event: one or more outcomes from an experiment

Outcome: the result of an experiment

Intersection: elements (parts) that are common to both sets

Union: the combination of elements in two sets

Expected Value: the value/ outcome that a prediction would suggest you will get

Universal Set: the set that has all the elements

Systematic: ordering values or outcomes with a strategy and sequence

Product: the answer when two or more values are multiplied together.

Add, Subtract and multiply fractions

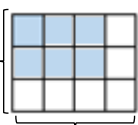
Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3} = \frac{12}{15} - \frac{10}{15} = \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

Multiplication

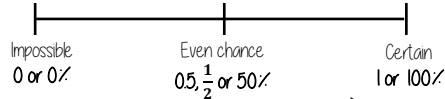
$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Modelled: 

Parts shaded: 6

Total number of parts in the diagram: 12

Likelihood of a probability



The more likely an event the further up the probability it will be in comparison to another event (it will have a probability closer to 1)

Sum to 1

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$
 \therefore The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability

What we expect to happen

Experimental probability

What actually happens when we try it out

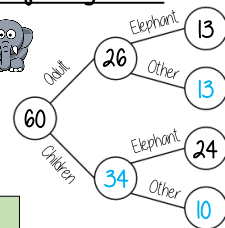
The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
 Theoretical probability is proportional

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adults' favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

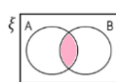
$$P(\text{adult}) = \frac{26}{60}$$

$$P(\text{Child with favourite animal as elephant}) = \frac{13}{37}$$

Two-way table

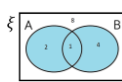
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



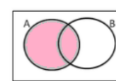
in set A AND set B

$$P(A \cap B)$$



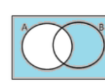
in set A OR set B

$$P(A \cup B)$$



in set A

$$P(A)$$



NOT in set A

$$P(A')$$

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

$$P(\text{Even number and tails}) = \frac{3}{12}$$

Independent events

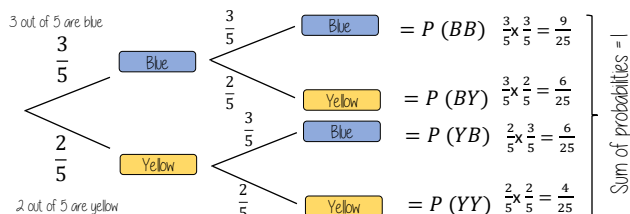
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$$P(A \text{ and } B) = P(A) \times P(B)$$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability

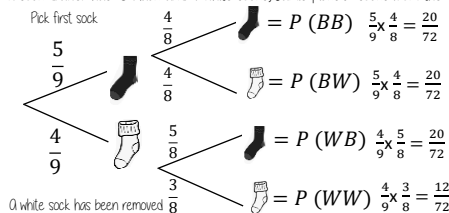


Dependent events

Tree diagram for dependent event

The outcome of the first event has an impact on the second event

A sock drawer has 5 black and 4 white socks. Jamie picks 2 socks from the drawer.



NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced \therefore the denominator is also reduced for the second pick.

YEAR 10 — DELVING INTO DATA...

@whisto_maths

Collecting, representing and interpreting

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

Population: the whole group that is being studied

Sample: a selection taken from the population that will let you find out information about the larger group

Representative: a sample group that accurately represents the population

Random sample: a group completely chosen by chance. No predictability to who it will include

Bias: a built-in error that makes all values wrong by a certain amount

Primary data: data collected from an original source for a purpose.

Secondary data: data taken from an external location. Not collected directly

Outlier: a value that stands apart from the data set

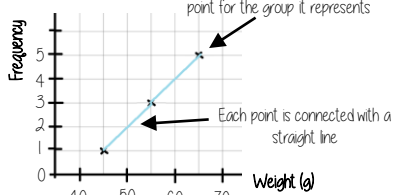
Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations

MID POINTS

Mid-points are used as estimated values for grouped data. The middle of each group

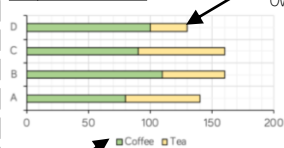


The data about weight starts at 40. So the axis can start at 40

Mid-point
Start point + End point
2

Bar and line charts

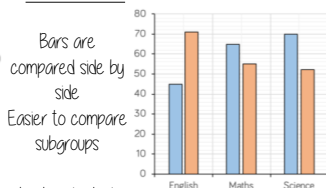
Composite bar charts



Categories clearly indicated

Compare the bars green compared to yellow. The size of each bar is the frequency. Overall total easily comparable

Dual bar charts



Bars are compared side by side. Easier to compare subgroups

Categories clearly indicated

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total: 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Two way tables

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adults' favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table

Subgroups each have their own heading			
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subarup totals

Overall total

Needs subgroup totals

Overall total

Draw and interpret Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$

"32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$



Use a protractor to draw. This is 192°

Multiple method

As 60 goes into 360 — 6 times. Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts

You NEED the overall frequency to make any comparisons

Averages from lists

The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

$55 \div 5$

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

Mode = 8

This can still be easier if the data is ordered first

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group — which group has the highest frequency

YEAR 10 — DELVING INTO DATA...

@whisto_maths

Collecting, representing and interpreting

What do I need to be able to do?

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Secondary data: data taken from an external location Not collected directly

Outlier: a value that stands apart from the data set

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

```

0 | 7 9
1 | 4 5 6 8 8
2 | 1 3
3 | 0
    
```

Key: 1 | 4 Means 14 years old

Stem and leaf diagrams:

Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

Girls	Boys
5	14
7, 5, 5, 5, 4	15 3, 8, 9
8, 4, 2, 1, 0	16 2, 5, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17 0, 2, 3, 6, 6, 7, 7
	18 0, 1, 4, 5

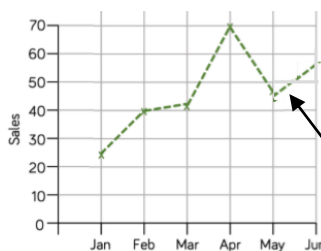
15 | 3,
Means 153 cm tall

Back to back stem and leaf diagrams

Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency

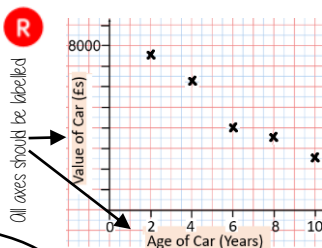
Mean, mode, median — allows for a comparison about more or less average

Range — allows for a comparison about reliability and consistency of data

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



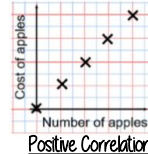
All axes should be labelled

The axis should fit all the values on and be equally spread out

"This scatter graph shows as the age of a car increases the value decreases"

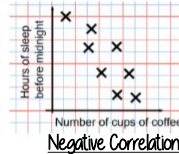
The link between the data can be explained verbally

Linear Correlation



Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases



No Correlation

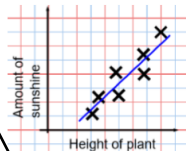
There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



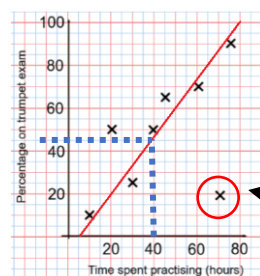
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful — in this example you cannot score more than 100%. So revising for longer can not be estimated

This point is an "outlier" It is an outlier because it doesn't fit this model and stands apart from the data

YEAR 10 — USING NUMBER...

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Non-calculator methods

What do I need to be able to do?

By the end of this unit you should be able to:

- Use mental/written methods for the four number operations
- Use four operations for fractions
- Write exact answers
- Round to decimal places and significant figures
- Estimate solutions
- Understand limits of accuracy
- Understand financial maths

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)

Round: making a number simpler, but keeping its place value close to what it originally was

Credit: money that goes into a bank account

Debit: money that leaves a bank account

Profit: the amount of money after income - costs

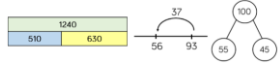
Tax: money that the government collects based on income, sales and other activities

Balance: The amount of money in a bank account

Overestimate: Rounding up — gives a solution higher than the actual value

Underestimate: Rounding down — gives a solution lower than the actual value

Addition/ Subtraction



Modelling methods for addition/ subtraction

- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative



$$6 + 3 = 3 + 6$$

The order of addition does not change the result

Subtraction the order has to stay the same

$$360 - 147 = 360 - 100 - 40 - 7$$

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
+	1	8	7
+	5	4	2

	H	T	O
-	4	2	7
-	2	4	9

Remember the place value of each column
You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

Short division $512 \div 7 = 73 \text{ R } 5$

Complex division

$$\div 24 = \div 6 \div 4$$

Break up the divisor using factors

$$3584 \div 7 = 512$$

Division with decimals

The placeholder in division methods is essential — the decimal lines up on the dividend and the quotient.

$$24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$$

Oil give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

	H	T	O
×	1	8	7
×	9		

Long multiplication (column)

Grid method

	H	T	O
×	1	8	7
×	9		

Repeated addition

Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
e.g. $0.2 \times 0.3 \rightarrow 2 \times 3$

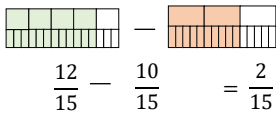
Make adjustments to your answer to match the question: $0.2 \times 10 = 2$
 $0.3 \times 10 = 3$

$$\text{Therefore } 0.2 \times 0.3 = 0.06$$

Four operations with fractions

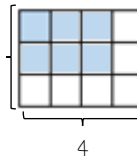
Addition and Subtraction

$$\frac{4}{5} - \frac{2}{3}$$



Multiplication

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$



Division

$$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

$$= \frac{8}{15}$$

Exact Values

Leave in terms of π

$$\frac{120^\circ}{360^\circ} \times 36\pi = \frac{1}{3} \times 36\pi = 12\pi$$

Leave as a surd



$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Estimation

Round to 1 significant figure to estimate

$$21.4 \times 3.1 \approx 20 \times 3 \approx 60$$

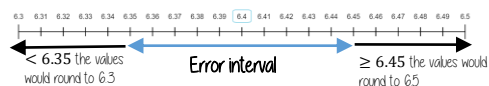
The equal sign changes to show it is an estimation

This is an underestimate because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths — it helps you identify calculation errors

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp.

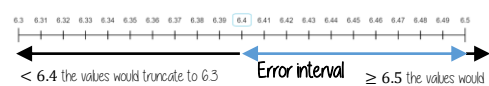


The error interval

$$6.35 \leq w < 6.45$$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp



$$6.4 \leq w < 6.5$$

Any value within these limits would truncate to 6.4 to 1dp

Rounding

2.46192 (to 1dp) - Is this closer to 2.46 or 2.47

2.46192

This shows the number is closer to 2.46

Significant Figures

- 370 to 1 significant figure is 400
- 37 to 1 significant figure is 40
- 3.7 to 1 significant figure is 4
- 0.37 to 1 significant figure is 0.4
- 0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

YEAR 10 — USING NUMBER...

Types of number & sequences

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What do I need to be able to do?

By the end of this unit you should be able to:

- Understand factors and multiples
- Express numbers as a product of primes
- Find the HCF and LCM
- Describe and continue sequences
- Explore sequences
- Find the n th term of a linear sequence

Keywords

Factor: numbers we multiply together to make another number

Multiple: the result of multiplying a number by an integer

HCF: highest common factor. The biggest factor that numbers share.

LCM: lowest common multiple. The first multiple numbers share.

Arithmetic: a sequence where the difference between the terms is constant

Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number

Sequence: items or numbers put in a pre-decided order

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15...

This list continues and doesn't end

$3x, 6x, 9x \dots$

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

45 is not a multiple of 3 because it is 3×15

Not an integer

Factors

Arrays can help represent factors

5×2 or 2×5

Factors of 10
1, 2, 5, 10

10×1 or 1×10

Factors and expressions

$x \ x \ x \ x \ x \ x$

$6x \times 1$ OR $6 \times x$

$x \ x$
 $x \ x$

$2x \times 3$

The number itself is always a factor

Factors of $6x$

$6, x, 1, 6x, 2x, 3, 3x, 2$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

The first prime number
The only even prime number

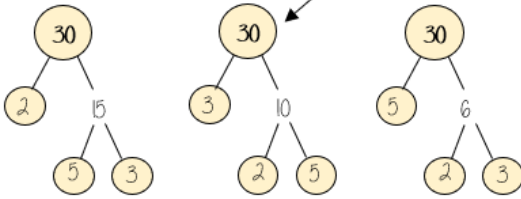
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Learn or how-to quick recall...

2, 3, 5, 7, 11, 13, 17, 19, 23, 29...

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

$30 = 2 \times 3 \times 5$

Multiplication of prime factors

Using prime factors for predictions

eg 60 30×2 $2 \times 3 \times 5 \times 2$
150 30×5 $2 \times 3 \times 5 \times 5$

Finding the HCF and LCM

HCF — Highest common factor

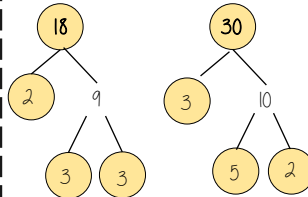
HCF of 18 and 30

18 1, 2, 3, 6, 9, 18

30 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share

HCF = 6



LCM — Lowest common multiple

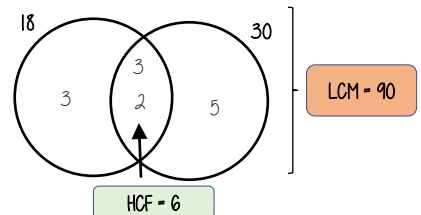
LCM of 18 and 30

18 18, 36, 54, 72, 90

30 30, 60, 90

The first time their multiples match

LCM = 90



Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found by multiplication/ division between terms

Term to term rule — how you get from one term (number in the sequence) to the next term

Position to term rule — take the rule and substitute in a position to find a term. Eg Multiply the position number by 3 and then add 2

Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 ...

Each term is the sum of the previous two terms

Triangular Numbers — look at the formation

1, 3, 6, 10, 15 ...

Square Numbers — look at the formation

1, 4, 9, 16 ...

Sequences are the repetition of a pattern

Finding the n th term

This is the 4 times table $\rightarrow 4, 8, 12, 16, 20 \dots$

$4n$

This has the same constant difference — but is 3 more than the original sequence

7, 11, 15, 19, 22

$4n + 3$

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

YEAR 10 — USING NUMBER...

@whisto_maths

Indices & Roots

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify square and cube numbers
- Calculate higher powers and roots
- Understand powers of 10 and standard form
- Know the addition and subtraction rule for indices
- Understand power zero and negative indices
- Calculate with numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers

Commutative: an operation is commutative if changing the order does not change the result

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Exponent: The power — or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent

Negative: A value below zero.

Coefficient: The number used to multiply a variable

Square and cube numbers

Square numbers

1, 4, 9, 16...

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$(2 \times 2 \times 3) \times (2 \times 2 \times 3)$$

12 x 12

Prime factors can find square roots

$$\sqrt{144} = 12$$

Cube numbers

1, 8, 27, 64, 125...

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$(2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

6 x 6 x 6

$$\sqrt[3]{216} = 6$$

Higher powers and roots

x^n — n — power (number of times multiplied by itself)

x — the base number

$\sqrt[n]{x}$ — Finding the n th root of any value

Other mental strategies for square roots

$$\begin{aligned}\sqrt{810000} &= \sqrt{81} \times \sqrt{10000} \\ &= 9 \times 100 \\ &= 900\end{aligned}$$

Standard form

Any number between 1 and less than 10

$$A \times 10^n$$

Any integer

$$\begin{aligned}0.001 &= 1 \times \frac{1}{1000} \\ &= 1 \times 10^{-3}\end{aligned}$$

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
10	1	0.1	0.01	0.001

Any value to the power 0 always = 1

Numbers in standard form with negative powers will be less than 1

$$3.2 \times 10^{-4} = 3.2 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 0.00032$$

Example

$$\begin{aligned}3.2 \times 10^4 &= 3.2 \times 10 \times 10 \times 10 \times 10 \\ &= 32000\end{aligned}$$

Non-example

$$\begin{aligned}(0.8) \times 10^4 &= 8000 \\ 5.3 \times 10^{07} &= 5.3 \times 10^7\end{aligned}$$

Negative powers do not indicate negative solutions

Addition/ Subtraction Laws

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

Zero and negative indices

$$x^0 = 1$$

$$\begin{aligned}\frac{a^6}{a^6} &= a^6 \div a^6 \\ &= a^{6-6} = a^0 = 1\end{aligned}$$

Negative indices do not indicate negative solutions

$$\begin{aligned}2^2 &= 4 \\ 2^1 &= 2 \\ 2^0 &= 1\end{aligned}$$

$$2^{-1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{4}$$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12} \leftarrow a \times b = 3 \times 4 = 12$$

NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Standard form calculations

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard form at the end

Method 1

$$\begin{aligned}6 \times 10^5 + 8 \times 10^5 &= 600000 + 800000 \\ &= 1400000 \\ &= 1.4 \times 10^6\end{aligned}$$

Multiplication and division

$$\begin{aligned}\frac{1.5 \times 10^5}{0.3 \times 10^3} &= \frac{1.5}{0.3} \times \frac{10^5}{10^3} \\ &= (1.5 \div 0.3) \times (10^5 \div 10^3) \\ &= 5 \times 10^2\end{aligned}$$

Method 2

$$\begin{aligned}6 \times 10^5 + 8 \times 10^5 &= (6 + 8) \times 10^5 \\ &= 14 \times 10^5 \\ &= 1.4 \times 10^1 \times 10^5 \\ &= 1.4 \times 10^6\end{aligned}$$

This is not the final answer

Division questions can look like this

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations